DESIGN METHODOLOGY FOR SATISFYING FLEET WEAPON SYSTEM PERFORMANCE REQUIREMENTS

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13. ABSTRACT

An analytical approach that can be used in the design of a weapon system fire control so as to ensure performance compliance for a given percentage of a fleet weapon system is described. Specifically, by taking into consideration the statistical nature of bias errors across an ensemble of weapon systems, it is shown that a rigorous analytical procedure exists for predicting fleet performance in terms of statistics associated with weapon system component bias and random errors.

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INTRODUCTION

The key measure of performance of a rapid-fire weapon system is the burst hit probability. It is this statistical quantity that is most often used to express performance requirements for the development of a new weapon system.

A desired level of performance, measured in terms of burst hit probability, is achievable provided bias and deterministic errors are maintained below some specified threshold and provided the ballistic dispersion, which accounts for the variation of the impact of the rounds about the aim-point, is held to some specified level.

The mathematical expression relating the burst hit probability to these error sources is well known and provides the fire control engineer with a powerful analytical tool with which to design his fire control. A significant part of the design process is to develop an error budget which imposes error tolerances for the design and manufacture of each of the error contributing elements of a weapon system so as to ensure that the weapon system meets the required level of performance. Implicit in the development of such an error budget is the assumption that every weapon system built to the tolerances established by the error budget meets the performance requirements when tested. Experience from field tests, however, demonstrates that such an assumption is unwarranted. Nor is it defensible on engineering grounds, for it is conceivable that given the statistical nature of the errors contributed by the various elements, these errors may combine, for some particular weapon system, in such a way that this system will fail to meet performance specifications. Clearly then, unless the performance requirements are extremely lax, it is impossible to design a weapon system for which compliance with requirements is achieved for 100% of the weapon systems in a fleet. For this reason, it is reasonable to require that statements of performance be expressed in terms of the percentage of a fleet of weapon systems expected to meet the stated requirements.

The purpose of this report is to advance an error budget development approach in which the percentage of fleet requirements is factored into the design process. Such an approach will typically result in an error budget that differs from that developed under the conventional method. Under the conventional approach, tolerances placed on the individual fire control elements are usually not sufficiently tight to allow for an acceptable fraction of the fleet to meet performance requirements. Furthermore, the two approaches produce significant differences in the value of dispersion that optimizes performance.

DEFINITIONS AND ASSUMPTIONS

In fire control, errors are generally classed according to their behavior over the duration of a burst, regardless of their origin or the statistical distribution to which they belong. They thus fall into one of three classes:

- Bias. These are errors that remain constant during a burst, but which typically vary from burst to burst and from one weapon system to another. Depending on the source, some bias errors may be controllable.

- Dispersion. These errors are random, and are responsible for the variation of the impact point of each round within a burst. Dispersion is due primarily to variations

in gun and ammunition characteristics as well as variations in the gun orientation due to recoil. A measure of the dispersion is usually available early in the development phase of the weapon system and is taken to be the same for all bursts and all weapon systems in a fleet. It may be possible to design for a particular value of dispersion that maximizes hit probability.

- Deterministic. These are a form of bias errors that remain constant during a burst and are the same for each weapon system. A prime example is target prediction error. Given an engagement scenario and a prediction algorithm, estimates of target prediction error are often attainable.

Throughout this report, random errors were assumed to be normally distributed. The notation $x \sim N(m_x, \sigma_x)$ is used to signify that x is a normally distributed random variable whose mean value is m_x and whose standard deviation is σ_x. Where appropriate, the symbol $\bar{x}$ will be used to designate the mean value of x. Unless otherwise stated, no correlation among errors is assumed.

BURST HIT PROBABILITY

The notion of burst hit probability is central to the design of a fire control system. It is used in the design of weapon systems to allocate an error budget among various fire control components; it is also a measure of system performance.

The determination of a burst hit probability requires the definition of a suitable coordinate system to which the target and the incoming rounds may be referenced. In what follows, a burst of N rounds fired at a target from a single weapon system shall be referenced to a two-dimensional rectangular coordinate system centered at the target and in a plane which is normal to the terminal portion of the trajectory. The x axis is horizontal and the y axis is normal to x. The ith round of an N round burst will then have coordinates x_i, y_i. In addition, it is assumed that each error source can be converted into a gun pointing error, expressed in milliradians, which, in turn, translates into a two component projectile miss-distance error, e_x, e_y. For example, an error in measuring the temperature of the propellant of a round translates to a miss-distance at the target measured in some convenient unit such as the meter or milliradian. Furthermore, the relation between the original error source and the resulting miss distance error is assumed to be linear so that the condition of normality, assumed for the original error source, holds for the miss distance error.

The total bias components of the miss distance error for any given round in a burst may be written as

$$m_x = b_x + D_x$$

and

$$m_y = b_y + D_y \tag{1}$$

where b_x, b_y are the two components of the bias error and D_x, D_y are the components of the deterministic error. It is assumed that for a rapid fire burst, the two components m_x, m_y remain

unchanged for each round in a burst. If e_{xi}, e_{yi} are the random components for the ith round, the components of the total error are then given by

$$E_{xi} = m_x + e_{xi}$$

and

$$E_{yi} = m_y + e_{yi} \tag{2}$$

The random component is measured from the centroid of the burst. Its mean value, taken about the center of the burst, is therefore zero. Thus assuming that N is large

$$\overline{E}_x = \overline{m}_x = m_x = b_x + D_x$$

and

$$E_y = \overline{m}_y = m_y = b_y + D_y \tag{3}$$

Taking the difference between equations 2 and 3, squaring, then taking the expected value, one finds the variance of the total error to be the variance of the random error e_x, e_y, or the dispersion. That is

$$\sigma_x^2 = \sigma_{dx}^2$$

$$\sigma_y^2 = \sigma_{dy}^2 \tag{4}$$

where σ_{dx}^2, σ_{dy}^2 are the variances of the dispersion along each of the two axes.

In terms of these quantities, the single shot hit probability is given by

$$P_{ss} = \frac{1}{2\pi\sigma_x\sigma_y} \int_A e^{-(x-m_x)^2/2\sigma_x^2} e^{-(y-m_y)^2/2\sigma_y^2} dA \tag{5}$$

where the integration is over the target area A. This value of single shot hit probability remains unchanged if the firing conditions don't change.

For a burst of N rounds, the burst hit probability is given by

$$P_b = 1 - (1 - P_{ss})^N \tag{6}$$

The firing conditions usually vary from one burst to the next, so the burst hit probability then also varies from burst to burst. This variation is manifest in the changing values of (m_x, m_y), the

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coordinates of the centroid of the burst. The usual assumption is that over all bursts fired by a single weapon system, the distribution of the m_x, m_y is Gaussian so the burst hit probability P_B averaged over all bursts is given by

$$P_B = \frac{1}{2\pi\sigma_{bx}\sigma_{by}} \int_{-\infty}^{\infty} P_b e^{-(m_x - \overline{m}_x)^2 / 2\sigma_{bx}^2} e^{-(m_y - \overline{m}_y)^2 / 2\sigma_{by}^2} dm_x dm_y \qquad (7)$$

with P_b in the integrand given by equation 6. The variances of the bias, σ_{bx}, σ_{by} are computed over all bursts.

In the conventional budget development approach, it is this relation that is used to establish the overall error tolerance of the weapon system. Furthermore, the inference is usually made that the values of σ_{bx}, σ_{by} apply across an ensemble of weapon systems.

The proper interpretation of the σ_b's as ensemble statistics and their relation to percentage of fleet requirements is developed in the following section. The objective of this section is to show how these values are delimited by weapon system performance criteria, thereby providing the fire control engineer the means to set tolerances on each of the element error sources that are constrained by these limiting values.

WEAPON SYSTEM ENSEMBLE

Consider a set of particular weapon systems originating from the same manufacturing run. If the number of weapon systems in the set is large, one may refer to these weapon systems as constituting an ensemble. However, each fire control element of an individual weapon system has an inherent measurement or performance error that may be either random or bias or both. If the error is a bias, then it is fixed for a given member of the ensemble. Across the ensemble, however, it is a random variable assumed to be normally distributed. Thus, a weapon system selected at random from the ensemble, is characterized by a particular value of gun/ammo dispersion, which is assumed to be the same for each member of the ensemble and, for a single burst, a fixed bias error that is made up of the algebraic sum of the effect of all the element bias errors in the system. Given the values of the dispersion and bias errors, as well as the specifics of the engagement scenario and burst size, one may then calculate the burst hit probability for that particular weapon system. If it is assumed that the bias error remains constant during the burst, then equations 5 and 6 apply.

Observe that in equation 5, m_x, m_y are fixed for a particular member of the ensemble. Geometrically, they represent the coordinates of the center of the burst. Selection of another weapon system from the ensemble will yield a different value of P_b since that system is characterized by a different value of the bias m_x, m_y. It is assumed that for each selection, the firing conditions are identical.

Now consider some particular value of burst hit probability, P_{bo}, as set forth in a performance requirement, or equivalently, some fixed value of single shot hit probability P_{sso}. The problem is to determine the fractional part of the ensemble for which $P_{ss} \geq P_{sso}$. The solution lies in the fact that the b_i $(i = x, y)$ and hence the m_i are normally distributed over the ensemble. Since $P_{ss} = P_{ss}(\sigma_d, m_i, A)$ with m_i given by equation 1 and A is the target area, and since both σ_d and A are fixed across the

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ensemble, the only ensemble variables are the m_i. Assuming that σ_d and A are known, there then exists a locus of points (m_x, m_y) in an m_x, m_y plane such that the equation $P_{ss} = P_{sso}$ is satisfied for each point of the locus. This locus is a closed curve in m_x, m_y space such that on the curve, $P_{ss} = P_{sso}$ whereas inside the region bounded by the curve, $P_{ss} > P_{sso}$. Furthermore, from the symmetry implicit in equation 5, it is clear that if the target area projected onto the x and y plane is symmetric about the x and y axes then this curve is also symmetric about the m_x and m_y axes.

Without further constraints, a closed form solution to this problem is impossible. That is, given a numerical value for P_{ss}, one cannot find an algebraic equation for the locus from equation 5. However, the problem is greatly simplified if one assumes that a) the target is circular and that b) the two components of the variance of the bias error are equal; that is, that $\sigma_{bx}^2 = \sigma_{by}^2$. If the requirements are given in terms of a square target, one may assume, without incurring significant loss in accuracy, the projected area to be a circle of area equal to the area of the square target. From equation 1, it follows that $\sigma_{mx}^2 = \sigma_{my}^2$. The locus of points for which $P_{ss} = P_{sso}$ is then a circle. Inside and on the boundary of this circle, $P_{ss} \geq P_{sso}$; outside, $P_{ss} < P_{sso}$.

Let m be the radius of this circle. Then

$$m_x^2 + m_y^2 \leq m^2 \tag{8}$$

defines the region within which $P_{ss} \geq P_{sso}$. If one assumes that the mean of the bias is zero across the ensemble, then $b_i \sim N(0, \sigma_{bi})$ and from equation 1 , $m_i \sim N(D_i, \sigma_{bi})$. The probability F that the pair (m_x, m_y) lies within the region given by equation 8 is then

$$F = \frac{1}{2\pi\sigma_b^2} \int_c e^{-(m_x - D_x)^2 / 2\sigma_b^2} e^{-(m_y - D_y)^2 / 2\sigma_b^2} dm_x dm_y \tag{9}$$

where the integration is over the area of the circle whose radius is m. This is also the fractional part of the weapon system ensemble that satisfies the requirement $P_{ss} \geq P_{sso}$, since each member of the ensemble is characterized by a unique pair of values (m_x, m_y).

In the special case where $D_x = D_y = 0$, a closed form solution for F is readily obtained. Changing to polar coordinates, equation (9 may then be written as

$$F = \frac{1}{2\pi\sigma_b^2} \int_0^m \int_0^{2\pi} e^{-r^2 / 2\sigma_b^2} r \, dr \, d\theta$$

or

$$F = \left(1 - e^{-m^2 / 2\sigma_b^2}\right) \tag{10}$$

from which one obtains

$$m = \sigma_b \sqrt{-2\ln(1 - F)} \tag{11}$$

Observe that m increases as F approaches one. This is to be expected since large m represents small burst hit probability values that an increasingly large percentage of weapon systems can meet or exceed.

RESULTS

For many weapon systems, the gun/ammo dispersion is to some extent controllable, and can be incorporated as a design feature of the weapon system. Ideally, that value is chosen which maximizes the performance of the system. For this reason, it is desirable to have some measure of the variation of the burst hit probability with dispersion for fixed values of F and σ_b. Armed with such information, the weapon system designer may then select that value of σ_d, which maximizes the burst hit probability for a given value of F.

Such information may be generated by computing m from equation 11 for given F and σ_b, then computing P_{sso} from equation 5 under the assumption that the dispersion along the two axes is equal. Assuming that the projected area of the target onto the x-y plane is a circle of radius R, P_{sso} is then constant everywhere on the circle $m_x^2 + m_y^2 = m^2$. One may then choose $m_y = 0$ in equation 5 so that $m_x = m$. Converting to polar coordinates with $x = r\cos\theta$, $y = r\sin\theta$, the integral for P_{sso} in equation 5 then becomes

$$P_{sso} = \frac{1}{2\pi\sigma_d^2} \int_0^R \int_0^{2\pi} e^{-(r\cos\theta - m)^2 / 2\sigma_d^2} e^{-r^2\sin^2\theta / 2\sigma_d^2} r\,dr\,d\theta \tag{12}$$

A tank size target is assumed in this analysis. Its dimensions are 2.3 m on a side. Converting this into a circular target of equal area, its radius is then 1.3 m. If a burst size of 20 rounds is assumed, one may then use equations 12 and 6 to generate the variation of P_b with σ_d. The results are shown in figures 1 through 3 for the case where the deterministic error is zero.

As is evident from figure 1, when the standard deviation of the bias is small, ~.5 milliradians, a large variation in dispersion can be tolerated with little or no affect on burst hit probability. In addition, the sensitivity of F to P_b is slight, so that when a given burst hit probability is met or exceeded by 80% of the weapon systems, 99% of the weapon systems will achieve nearly the same level of performance. However, when the bias is 1 milliradian or larger, (figs. 2 and 3), the sensitivity of burst hit probability to dispersion is pronounced; so is the burst hit probability to the percentage F of weapon systems that meet or exceed a specific level of performance.

Figure 4 was generated for the case where the deterministic error D has a non-zero value. To generate the curves in this figure, one must first note that the center of the bivariate normal distribution in equation 9 is at (D_x, D_y) and that this distribution is circularly symmetric about this point. The existence of this symmetry is most readily demonstrated by noting that for the case where $D_x = D_y = 0$, the integrand possesses circular symmetry (eq 10). Non-zero values of D_x, D_y merely translate this two-dimensional distribution to a new location centered at (D_x, D_y) so that circular

symmetry is preserved about this point. It then follows that since the integration is over a circle, F depends only on the distance $D = \sqrt{D_x^2 + D_y^2}$ and not on the orientation of the vector $\vec{D}$. For the case where $D_x = D_y = 1$, $D = |\vec{D}| = \sqrt{D_x^2 + D_y^2} = \sqrt{2}$. By these arguments, one is free to choose $D_y = 0$ so that $D_x = D = \sqrt{2}$ and for $\sigma_b = 1$, equation 10, written in polar coordinates becomes

$$F = \frac{1}{2\pi} \int_0^m \int_0^{2\pi} e^{-(r\cos\theta - \sqrt{2})^2/2} e^{-(r\sin\theta)^2} r\,dr\,d\theta \tag{13}$$

Unlike equation 10, equation 13 does not admit a closed form solution. However, one may obtain numerical solutions to F by assigning numerical values to m. Choosing the desired values of F and the corresponding values of m, one then uses those values of m in equation 5 which, together with equation 6, can be used to generate the curves in figure 4.

The presence of a non-zero deterministic error degrades performance as can be readily noted by comparing figures 4 with 2. Furthermore, dispersion must be increased in order to maximize hit probability.

Observe that for a given F, there exists some value of σ_d that maximizes P_b. The objective of a good fire control design is then not to minimize the dispersion, but to select that value, based upon knowledge of bias and deterministic errors, that will maximize burst hit probability.

As expected, the maximum value of P_b increases as F decreases. That is, a lower percentage of weapon systems can achieve higher hit probability values. Table 1 lists those values of σ_d which maximize P_b for the various combinations of F and σ_d that appear in figures 1 through 4. The value of m corresponding to a given F is also listed. Under the conventional approach, one would use equation 7 to generate P_b versus σ_d. A sample plot is shown in figure 5, where it is assumed that there is no deterministic error and that the dispersions along the two axes are equal. The means of the bias errors $\overline{m}_x$, $\overline{m}_y$ over all bursts fired are also assumed to be zero.

The assumption usually made, when applying P_b versus σ_d curves such as the one in figure 5 to a fire control design, is that they represent the performance of each member of a fleet of weapon systems. As noted earlier, such an assumption is fallacious because it does not take into account the statistical properties of bias errors across an ensemble of weapon systems. As examination of figures 2 to 4 reveals the larger the percentage of weapon systems that is expected to meet requirements, the lower the expected value of burst hit probability.

If one were to design a system with a $\sigma_b = 1$ milliradian and a dispersion of 1.4 milliradians, then, according to figure 5 (also table 1), a maximum probability of 0.975 is achievable. However, as figure 2 indicates, only about 80% of the fleet would achieve this value were these design parameters to be used. This example illustrates how, using the conventional approach, prediction of system performance is overly optimistic and may lead to acceptance of error budget tolerances that could result in unacceptable performance for a sizable percentage of the fleet.

It should be noted that although the emphasis in this report has been on burst hit probability, the analysis is equally valid for burst kill probability. One need merely replace the target area with the target vulnerable area.
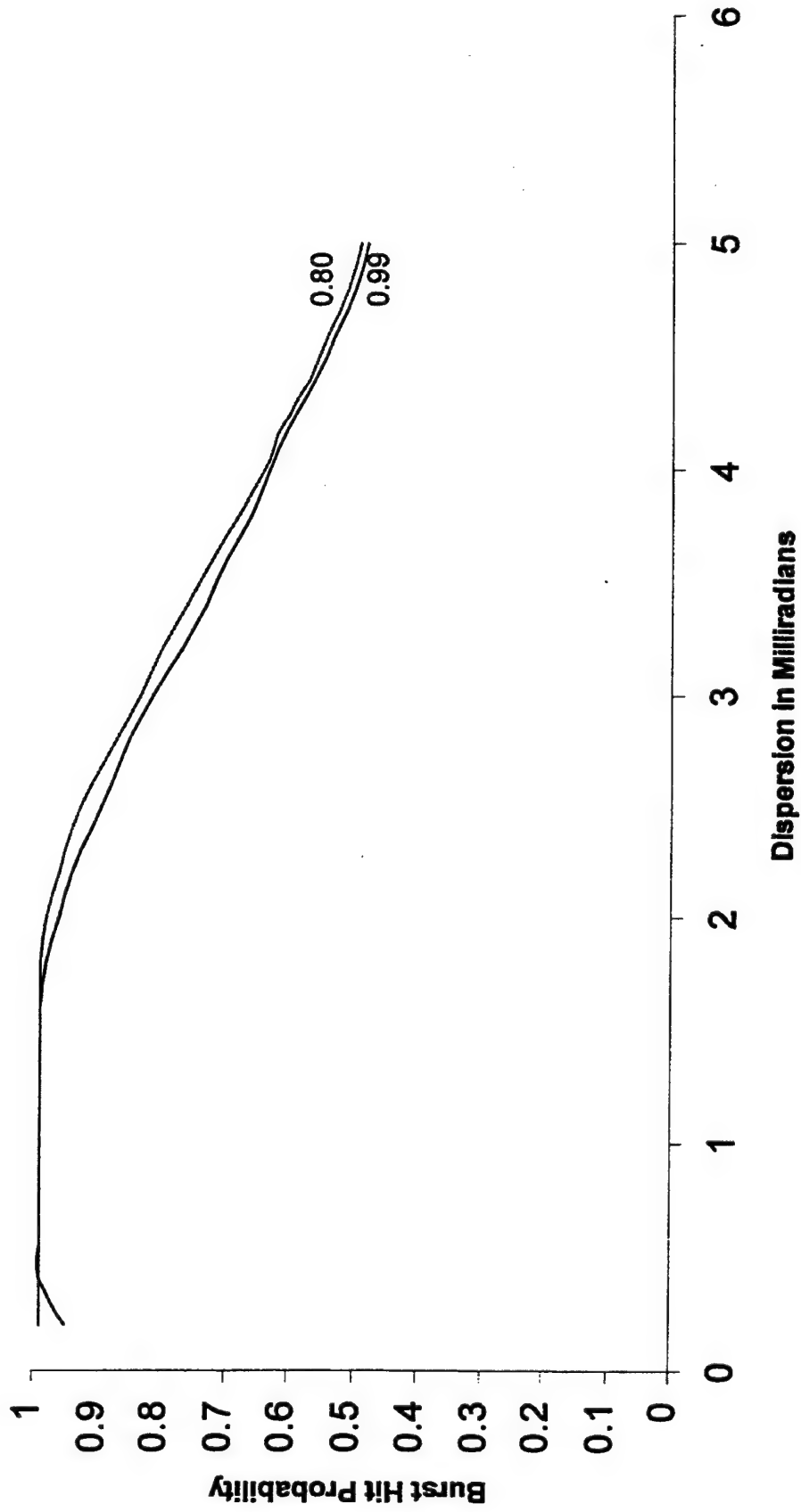
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Figure 1

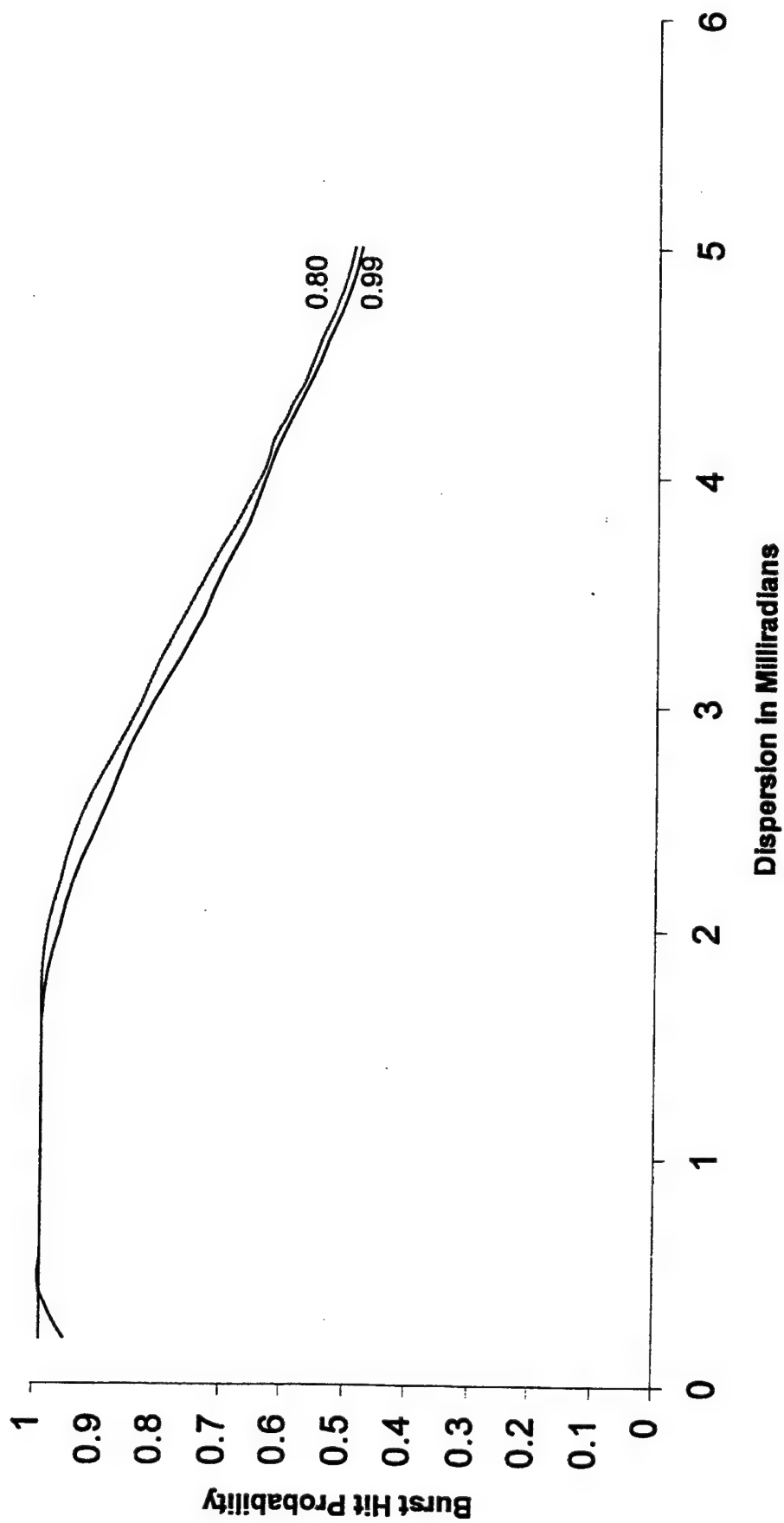Burst hit probability versus dispersion for σ_b = .5 milliradians

Figure 2

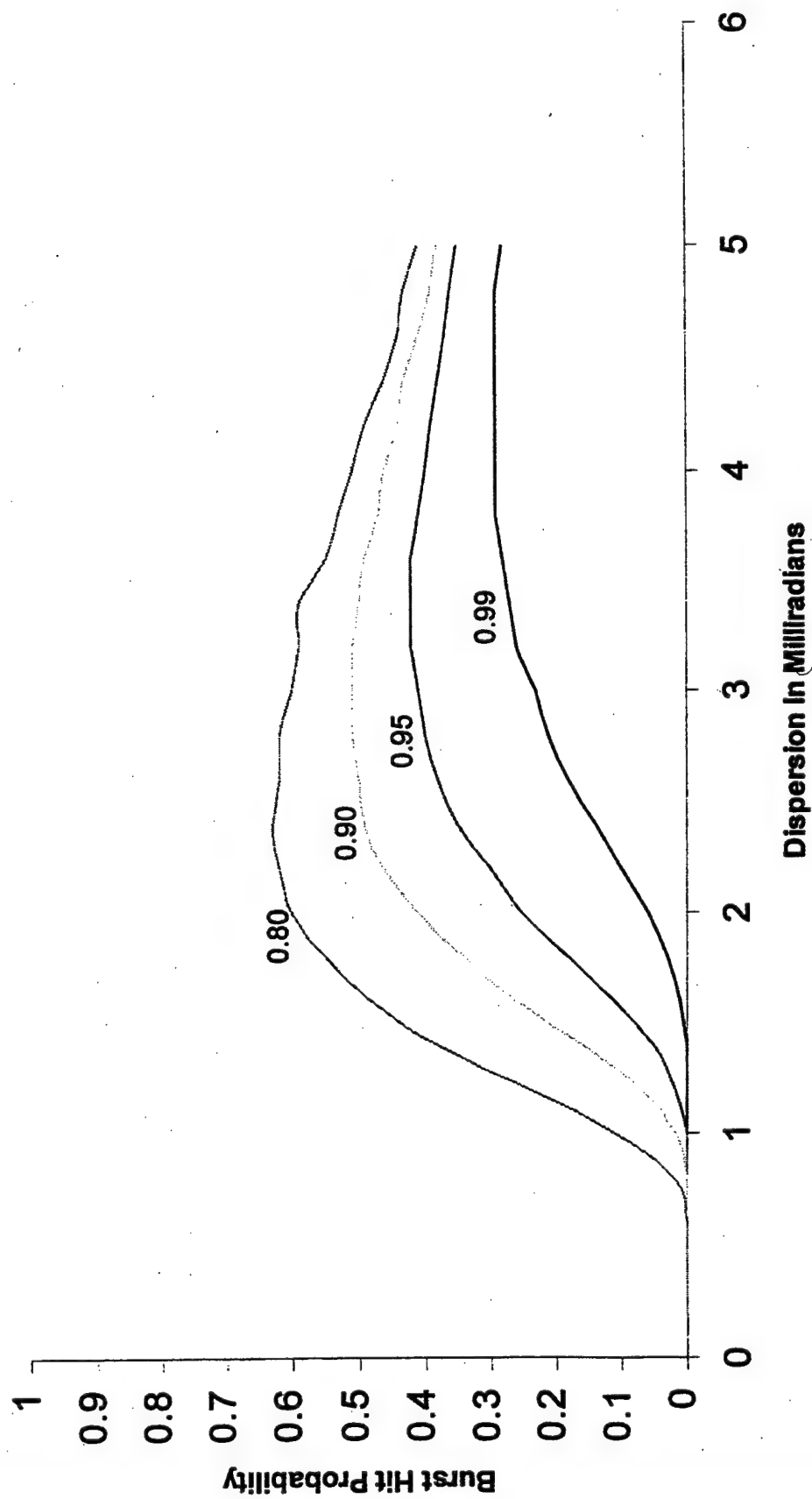Burst hit probability parameterized by F for σ_b = 1 milliradian

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Figure 3
Burst hit probability parameterized by F for σ_b = 2 milliradians
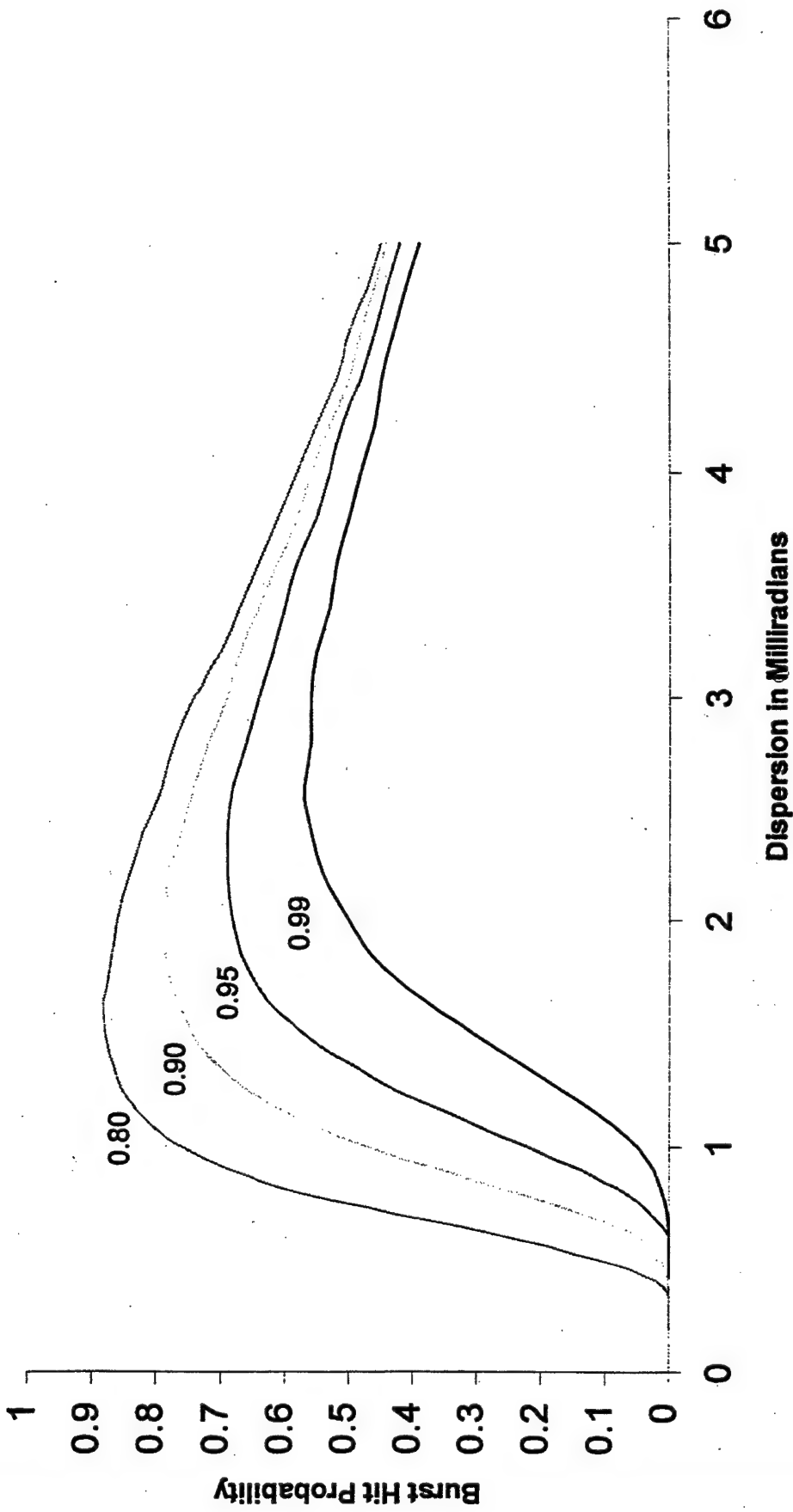
Figure 4
Burst hit probability parameterized by F for $\sigma_b = 1$ and $D_x = D_y = 1$
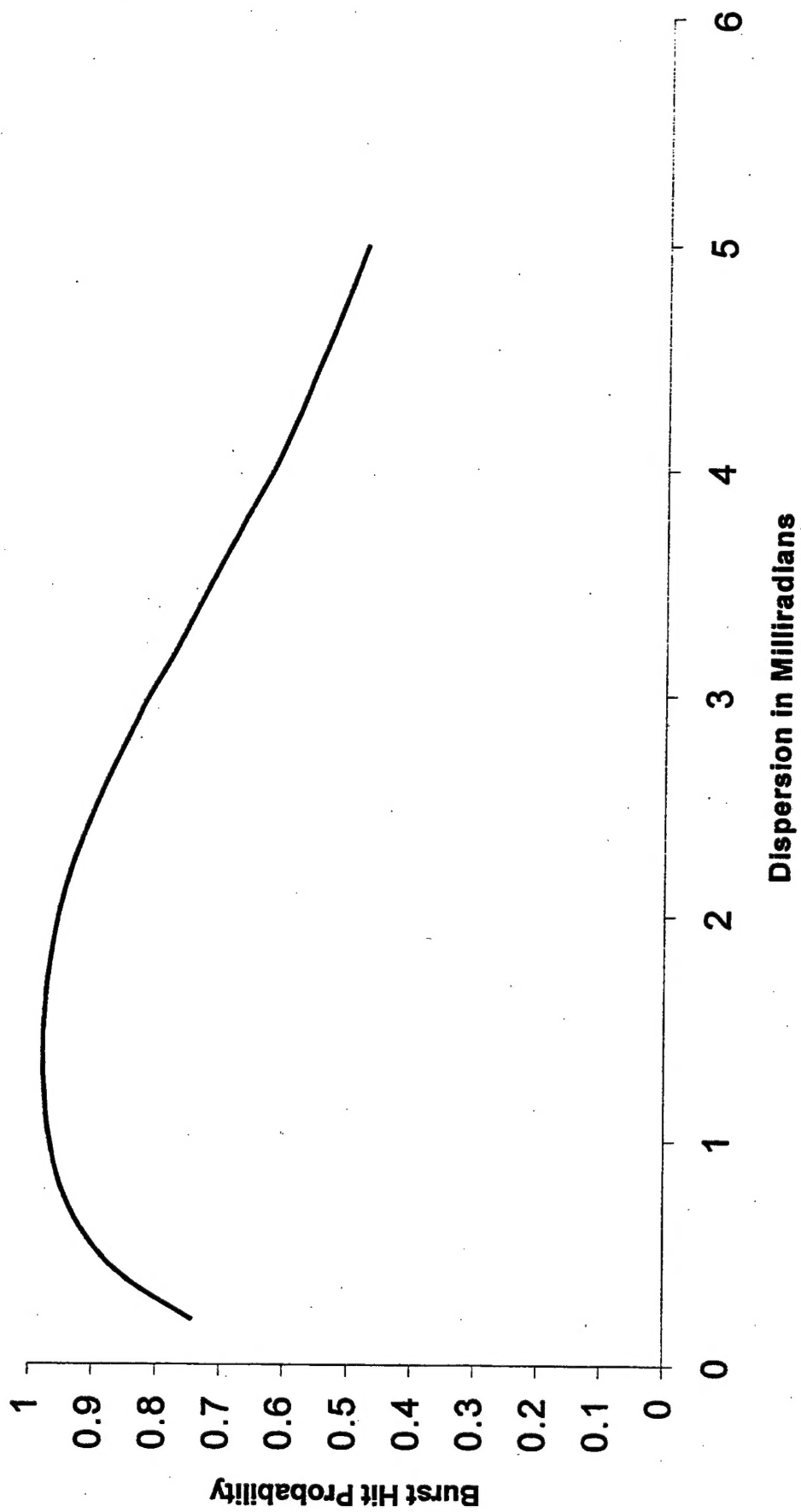
Figure 5

Variation of burst hit probability with dispersion for σ_b = 1 milliradian

Table 1
Values of dispersion σ_d which maximize P_b

$\sigma_b = .5 \quad D = 0$

F	m	σ_d	P_b
.80	.90	.60	.99
.99	1.5	1.0	.99

$\sigma_b = 1.0 \quad D = 0$

F	m	σ_d	P_b
.80	1.8	1.0	.99
.90	2.1	1.4	.95
.95	2.4	1.6	.90
.99	3.0	2.0	.76

$\sigma_b = 1.0 \quad D_x = D_y = 1$

F	m	σ_d	P_b
.80	2.5	1.6	.88
.90	2.9	2.0	.78
.95	3.3	2.2	.69
.99	3.9	2.6	.57

$\sigma_b = 2.0 \quad D = 0$

F	m	σ_d	P_b
.80	3.6	2.4	.63
.90	4.2	2.8	.51
.95	4.8	3.6	.42
.99	6.0	4.2	.29

$\sigma_b = 1.0 \quad D = 0$ for figure 5.

σ_d	P_b
1.4	.975

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